Novel Geometry and Symmetry in String Theory

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Colloquium at NTU, 11/06/2013

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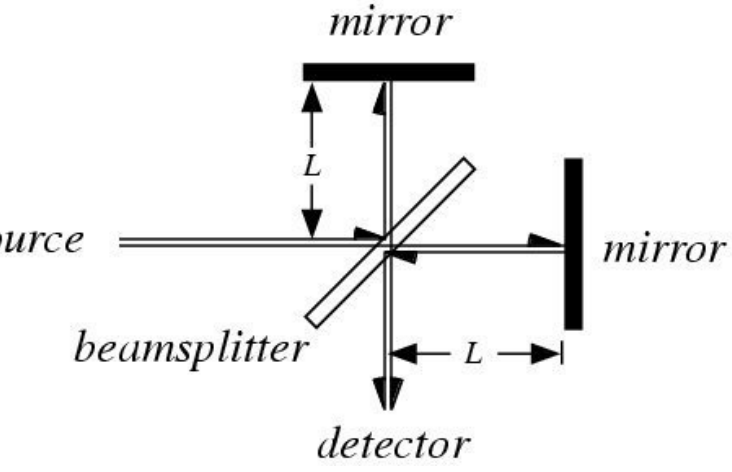
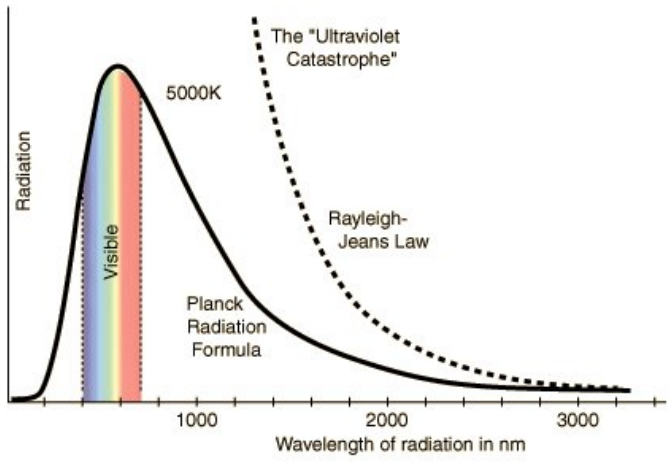
Two Clouds of Theoretical Physics in 1900





Two Clouds of Theoretical Physics

In 1900, in a lecture titled "Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light, Lord Kelvin, an influential British physicist, famously proclaimed that physics was over, except for two small clouds on the horizon.

These "clouds" turned out to be the clues that led to the discovery of quantum mechanics and special relativity.

- Lesson; One should not ignore any crash, especially if it is a theoretical one.
- One can always learn something in resolving the conflict! e.g. special relaivity; quantum mechanics; antimatter; UV divergeneces in QED; ...

Standard model of elementary particles and Einstein theory of gravity describe the world pretty well. However,

> Standard model is treated using the framework of Quantum field theory. However Einstein gravity is a classical theory and cannot be quantized using the framework of QFT.

Quantization of gravity remains one of the most important problem of theoretical physics.

Basics of String Theory

String theory is a simple modification of QFT:

- QFT = special relativity + quantum mechanics + point-like particles
  String Theory = special relativity + quantum mechanics + strings
- In addition to translation, string is also capable of vibration
- There is a unique fundamental object (i.e. string). Different vibrational mode of the elementary string give rises to particle excitation that has mass and spin depending on the vibrational mode.

One of the string excitations is a spin two massless particle and has the right properties of the gravition.
Scherk and Schwarz proposed in 1974 to use string theory to quantize gravity and to unify all forces.

Therefore, string theory provides a theory of quantum gravity.
In addition,
String Theory unifies matters and interactions!!

First string revolution (1984)

The original bosonic string theory suffered from a number of problems: It has no fermions. It lives in 26 dimensions and it has a tachyon in its spectrum.

These problems were resolved with the introduction of two crucial concepts:

i. Supersymmetry: supersymmetry introduces fermions and removes the tachyon.
ii. Compactification: superstring lives in 10 dimensions. The extra 6 spatial dimensions can curl up and become very small in a gravity theory. Most importantly this is a dynamical process in string theory and do not needed to be put in by hand.

As a result, five consistent superstring theories were found:

Type I, Type IIA, Type IIB, HE, HO

(Heterotic string with $E_8 \times E_8$ and $SO(32)$ gauge group)

These string theories live in 10 dimesnional spacetime and has only two parameters:

mass parameter α' (string tension);
string coupling constant g.

- Each of these superstring theory respects different supersymmetries and gauge symmetries.

However there are at least two unsatisfactory aspects:

Q1. Uniquness of String Theory

Why so many? Which is the one explaining our world?

Q2. Uniquness of the Vaccum

There are many ways one can compactify the extra dimensions. Why is a particular one choosen?

Some of these questions got answers in the last decade due to a number of advances in our understanding of the geometry and symmetry of string theory. Suprisingly, it's realized now:

String theory is not just a theory of strings.

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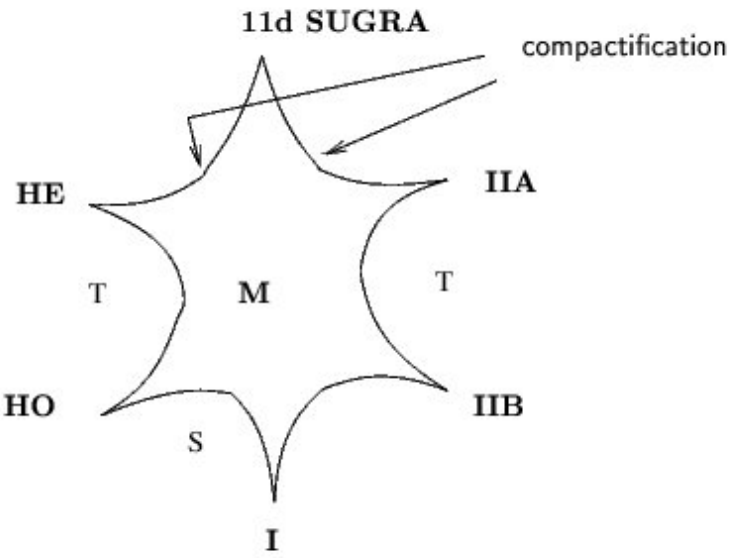
String duality

- It was believed that there were five distinct superstring theories: **I, IIA, IIB, HO, HE**.
  But only one of them is the actual correct describing the world. The other four would be mathematically consistent, but not used by nature (similar to gauge theory).
- Now it is known that this naive picture is wrong, The five superstring theories are connected to one another by transformations called duality transformations.

The five superstrings are considered to be different limits of the 11-dimensional M-theory.



This marks the begining of the second string revolution.

Witten (1995)

D-branes play very important roles in the establishment of this web of duality.

D-brane

A Dp-brane is an extended object (with p spatial dimensions and one time) upon which open strings can end on them. Its physics is completely determined by the dynamics of the open string.

Polchinski (1995)

The theory on the worldvolume of a single D-brane is an abelian gauge theory with a number of scalars and fermions arranged in a supersymmetrical way. E.g. for a D3-brane,

$$(A_\mu, \Phi^I), \qquad \mu = 0, \cdots, 3; I = 4, \cdots, 9$$

If we have a set (say N) of overlapping D3-branes, then the $U(1)$ theory get enhanced to a non-abelian $U(N)$ supersymmetric gauge theory:

$$(A_\mu^{ab}, \Phi^{I\,ab}), \qquad a, b = 1, \cdots, N$$

Witten (1996)

BFSS matrix model

- BFSS matrix model was originally proposed as a definition of M-theory: it was argued that the dynamics of the M-theory is captured by the physics of D0-branes.

  Banks, Fishler, Shenker, Susskind (1996)

- In the sector of N D0-branes, the action is given by the maximally supersymmetric $U(N)$ quantum mechanics:

$$S = \int dt \left[\sum_I (D_t X^I)^2 - \sum_{I,J} [X^I, X^J]^2 + \text{fermions} \right]$$

  Here the variables X^I are $N \times N$ unitary matrix, $I = 1, \cdots, 9$, and denotes the space coordinates as seen by N $D0$-branes.

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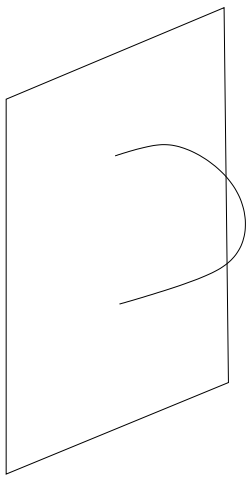
A novel aspect of geometry is the discovery of noncommutative geometry in string theory.

The worldvolume of a D-brane become noncommutative when a constant NSNS B_{ij}-field is turned on:

Noncommutative geometry: $[X^i, X^j] = i\theta^{ij}$.

Connes, Douglas, Schwarz (1997); Douglas, Hull (1997); Chu, Ho (1998); Schomerus (1999); Seiberg, Witten (1999)

- This result can be derived by quantizing open string in a B-field.



Chu, Ho (1998)

- It was discovered recently that a similar phenomena occurs for the M5-brane in the presence of a constant 3-form C-field potential

$$[X^i, X^j, X^k] = i\theta^{ijk}$$

Chu, Sembi (2010)

where

$$[f, g, h] := fgh + ghf + hfg - fhg - gfh - hgf,$$

is the quantum Nambu bracket (Nambu (1973)) defined on ordinary operators.
This is called the Quantum Nambu Geometry.

- The construction of QFT on Quantum Nambu geometry is potentially a very important topic both theoretically and phenomenologically.

Outline

A very fundamental question (perhaps crazy) of theoretical physics is about the nature of spacetime. What exactly is spacetime? Why is spacetime 4-dimensional? Why is there a time? Is spacetime a fundamental or an emergent/derived concept?

- In string theory, or its eleventh dimensions completion M-theory, allowable spacetime is determined dynamically as solution to the equation of moton.
- EOM of the BFSS theory is:

$$D_t^2 X^I - [X^J, [X^I, X^J]] = 0$$

- It has the time independent soln (exists only in the large N limit)

$$[X^i, X^j] = i\theta^{ij}\mathbf{1}, \quad \theta^{ij} = \text{constant}$$

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- To answer this question, we need to analysis the physics of the fluctuation around the background solution $[x^i, x^j] = i\theta^{ij}\mathbf{1}$. Observe that

$$x^i \sim i\theta^{ij}\partial_j$$

- The fluctuation around the solution to the EOM can be written as

$$X^i = x^i + A^i \sim \partial_i + A_i = D_i$$

where A_i describes the fluctuation over the background.

- Using

$$[X^i, X^j] \sim [D_i, D_j] = F_{ij}$$

$$[X^i, \psi] \sim D_i\psi$$

$$[X^i, X^a] \sim D_i X^a$$

As a result,

$$S_{BFSS}(X = x + A) = S_{NCSYM}(A),$$

This is precisely the low energy theory on the worldvolume of a Dp-brane with a NSNS B-field turned on. Seiberg (2000)

- In this example, spacetime and its properties (dimensions and existence of noncommutativity) is obtained dynamically!
- There has also been some analysis based on Monte Carlo suggesting that 4 dimensional spacetime is energically favorable over other dimensionality.
- This emergence of spacetime from a matrix model is very much in line with the philosphy that spacetime is a derived concept. However it is not understood how gravity arises in this picture.

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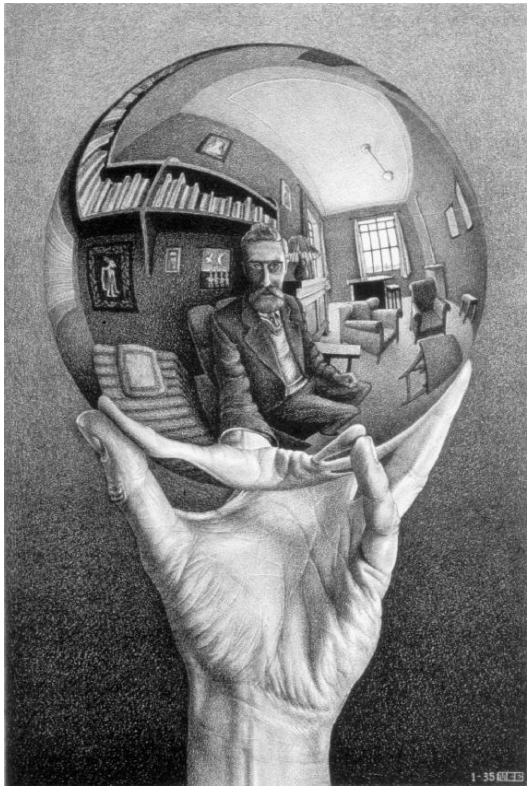
Area law of black hole entropy

$$S = \frac{A}{4}$$

leads to the Holographic Hypothesis: ('t Hooft, Susskind)

- Gravity in some region of space is completely described by a set of fundamental degree of freedom living on the boundary of the region.

According to holography, quantum gravity, that is spacetime itself and together with its quantum properties, are derived from a set of more fundamemtal degree of freedom living on the boundary.



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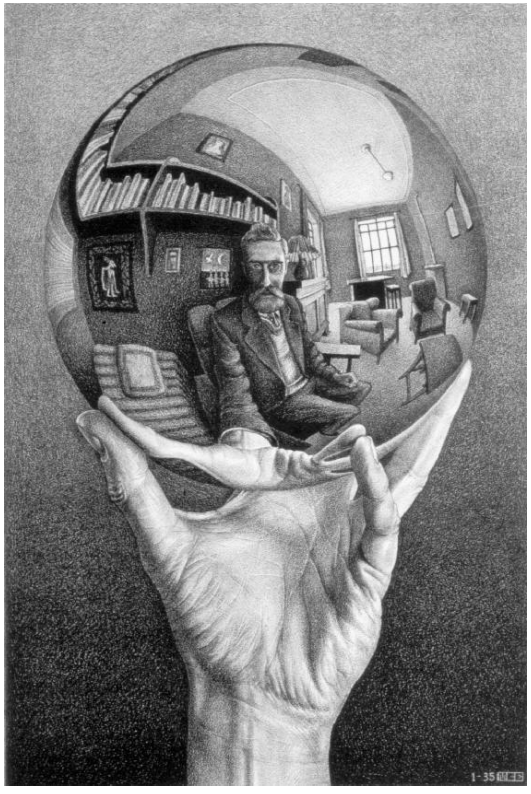

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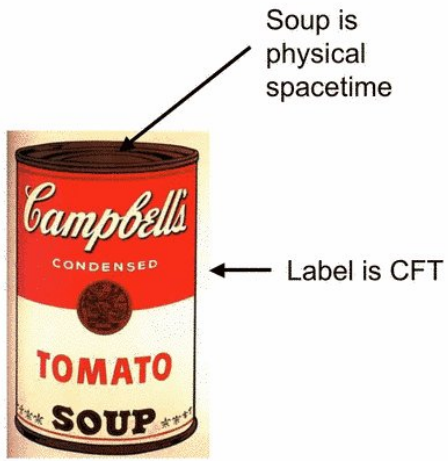
Realization of Holography: AdS/CFT correspondence

Maldacena (97) argued that:

- String theory on spacetimes which asymptotically approach $AdS \times X$
  $=$
  CFT living on the boundary at inifnity.



- *AdS: Anti de Sitter spacetime - the maximally symmetric spacetime with negative curvature*
  CFT: Conformal Field Theory - ordinary nongravitational Yang-Mills gauge field theory which is conformally invariant.

- This duality was derived from a decoupling limit of the near horizon physics of D3-branes.
  So far there is no proof, but lot of evidences supporting it.
- And there has been many interesting applications:
  i. AdS/plasma: properties of high temperature nuclear matter: quark/gluon plasma,
  ii. AdS/CDM: condensed matter physics
  iii. AdS/nuclear: nuclear physics
  ...

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- When N D-branes are put together on top of each other, the gauge symmetry is enhanced from $U(1)$ to $U(N)$:

$$\delta A^a_\mu = \partial_\mu \Lambda^a + [A_\mu, \Lambda]^a, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + [A_\mu, A_\nu]^a.$$

- M-theory contains M5-branes with a 2-form gauge field potential. For a single M5-brane, there is a $U(1)$ 2-form gauge field potential $B_{\mu\nu}$ living on the worldvolume. When N M5-branes are put together, one expect to have an enhancement of the gauge symmetry.

How?

- The construction of the gauge syjmmetry turns out to be very difficult. For example, it is nontrivial (open) to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B^a_{\mu\nu} = \partial_\mu \Lambda^a_\nu - \partial_\nu \Lambda^a_\mu + (?), \quad H^a_{\mu\nu\lambda} = \partial_\mu B^a_{\nu\lambda} + \partial_\nu B^a_{\lambda\mu} + \partial_\lambda B^a_{\mu\nu} + (?).$$

  to have nontrivial self interaction.

- Turns out the non-abelian terms "(?)" have to be non-local. The following generalized gauge symmetry has been proposed:

$$\delta B^a_{\mu\nu} = D_\mu \Lambda^a_\nu - D_\nu \Lambda^a_\mu, \quad D_\mu = \partial_\mu + A_\mu$$

  where A_μ is an auxillary gauge field determined entirely by the H field strength.

  (Chu 2011; Chu and Ko 2012)

  of M5-branes and this is

- The equation of motion for the multiple M5-branes is the self-duality equation:

$$H = *H$$

generalizing the self-dual Yang-Mills instanton equation $F = *F$).

- The instanton equation is very important both physically and mathematically. Moreover, it is exactly solvable via the ADHM (Atiyah-Drinfeld-Hitchin-Manin) construction.
- There are some evidences that it could be the same for the self-duality H-equation.

(Chu 2011; Chu withexit Ko, Vanichchapongjaroen, Isono 2012, 2013)

Outline

Main Lessons

- There has been lot of progress in the understanding of geometry and symmetry in string theory! There has also been many useful mathematical spinoff.
- Despite 40 years of effort, we are still far from having a complete satisfactory formulation of string theory.
  Carl sagan: Remarkable claims require remarkable proof!
  One may recall the development of quantum mechanics:

  1859: Kirchhoff's statement of the blackbody radiation problem
  1900: Planck formula $E = h\nu$.
  1924: de Brogile put forward theory of matter waves
  1925: Schrdinger equation and Heisenberg matrix mechanics
  1927: Heisenberg uncertainty principle

  It tooks many years to get to the modern formulation of quantum mechanics from the old quantum theory.

- It is likely that many results we have today may turn out to be too naive. But hopefully the lessons we learnt would guide us to the right principles for the formulation of the Theory of quantum gravity and Unification.
- It is quite amazing that string theory provides a framwork in which some of the most fundamental questions (e.g. structure of spacetime) may be addressed.
  String theory enables us to ask many questions that one cannot even dream of before.

But have we asked the right questions?